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Table of Contents

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Wu Lianda   Jia Peizhang

ABSTRACT

This article discusses in detail deletion problems associated with outliers in optical observations of artificial satellites. We first of all take complex nonlinear problems, making use of initial orbits and turn them into linear models convenient for digital processing. Use is made of two quantities--equations of time ΔT as well as orbital plane normal deviation $\Delta\theta$ in order to analyze and make determinations. In conjunction with this, a type of L-estimation target function opting for the use of trimmed least squares is put forward to carry out methods associated with deletion of outliers. The breakdown point associated with these methods is 50%. At the same time, computational examples are given. In conjunction with this, discussions are carried out with regard to initial orbit accuracies and deletion criteria.

KEY WORDS Satellite observation Outlier Robust estimation

In this, R is survey station geocentric range:

R is survey station geocentric vector;

Z = 90°-h is satellite zenith range;

l is satellite obervation direction, and

$$l = R(z,180^\circ - S)R(y,90^\circ - \varphi)l_0, \quad l_0 = (\cos h\cos A, -\cos h\sin A, \sin h)^\tau, \tag{2}$$

Here, S is survey station local sidereal time;

ϕ is survey station latitude.

Use r to carry out coordinate transformation. Take coordinates and transform them into a right hand coordinate system which is composed of xyz axses with an x axis pointing toward the orbital ascending node, and a z axis pointing in a direction normal to the orbital plane. One then has:

$$r_N = R(x,i)R(z,\Omega)r, \tag{3}$$

R(x,θ), R(y,θ), R(z,θ) in equation (2) and equation (3) are coordinate rotation transformation matrices (expressions omited). i and Ω are satellite angle of inclination and ascending node longitude.

Assuming

$$r_N = (x,y,z)^\tau, \quad u_0 = tg^{-1}(y/x), \tag{4}$$

one then has

Here, uo is the satellite longitude angle corresponding to r. This is different from u corresponding to time t radicals E. On the basis of binary problem formulae, it is not difficult for us to calculate the satellite level longitude λo . Then,

$$\Delta T = (\lambda_0 - \lambda)/n, \tag{5}$$

In this,

$\lambda = M + \omega$ is satellite level longitude associated with time t.

n is satellite level motion $n = \sqrt{\mu/a^3}$ and μ = GM.

Besides this,

$$\Delta\theta = \sin^{-1}(z/r) \tag{6}$$

ΔT and Δθ defined in equations (5) and (6) are different from the observation quantities A and h. They already correspond to satellite geocentric movements. Speaking in terms of observations

associated with a station ring (short arc), so long as initial orbital radicals are not very wrong, they can then be expressed with the use of linear models:

$$\Delta T_i = a_0 + a_1(t_i - t_0) + \zeta_i, \\ \Delta\theta_i = b_0 + b_1(t_i - t_0) + \eta_i, \quad (i = 1,2,\cdots\cdots N), \tag{7}$$

In this, ξi and ηi correspond to observation errors. We assume that they are zero mean values--mutually independent random values associated with normal distributions.

If we are able to precisely estimate ao, a1, bo, and b1, it is then possible to make use of the magnitude of $|\Delta T_i$ - ao - a1(t1 - to)$|$ $|\Delta\theta_i -$ bo - b1 (t1 - to)$|$ as well as the magnitudes of a1, bo, and b1 themselves in order to carry out outlier deletion. As a result, problems associated with outlier identification and deletion can be summarized as: already knowing measurement values {ΔTi} and {Δθi} (i=1,2....N), as far as certain outlier values contained within them are concerned, how to solve for robust estimations of ao, a1, bo, and b1. /115

II. ROBUST ESTIMATION METHODS

Assume that the already known quantities {yi}, (i = 1,2,...N), obey the linear model described below:

$$y_i = a_0 + b_0(t_1 - t_0) + e_i, \tag{8}$$

In the equation, {ei} (i = 1,2,....N) are observation errors. With regard to good observation quantities, {ei} can be assumed to be a zero mean value. Mutually independent random quantities associated with normal distributions have variances of σ_0^2. When {yi} has certain outliers in it, the robust estimation methods associated with ao and bo are very numerous [1-5]. We opt for the use of L estimation methods associated with trimmed least squares. The basic idea is to make target function:

$$U = \sum_{i=1}^{N\beta+1} (r_{Ni})^2 = \min, \tag{9}$$

In this, Nβ is a whole number (1/2≤β<1). (rN1) represents residual error sequences. It is possible to demonstrate that the estimates in question are consistant estimations. The breakdown point is ε* = 1 - β[3]. If we select β = 1/2, then the breakdown point is 50%.

If we are able to sample and obtain a pair of good observation quanitites yi, yj, then:

$$\left.\begin{array}{l} a_{ij} = (y_i + y_i)/2, \\ b_{ij} = (y_i - y_j)/(t_i - t_j) \end{array}\right\} \qquad (10)$$

This is nothing else than one type of relatively reasonable estimation of ao and bo. Because of this, the problem is summarized as: how to carry out sampling to make it possible to at least be able to obtain a set of good observation quantities. The clumsiest method is to take N pieces of data two by two in pairs.

It is possible to obtain C_N^2 individual samples. It is then possible to obtain C_N^2 individual target functions. If there are M pieces of good data in N pieces of data (M > N/2), then, in C_N^2 individual target functions, we can have C_M^2 individual relatively good estimations. Making target functions U adopt minima is nothing else than robust estimations of ao and bo associated with our final solution.

In order to save on calculations, we certainly do not need to calculate C_N^2 individual target functions. Our method is:

Let

$$m = \begin{cases} 1, & N \leqslant 10, \\ INT(N/5), & N > 10, \end{cases}$$
$$K = INT(N/m),$$

Taking each individual datum and each interval m individual datum which follows it and mating them up two by two, the number of iterations of calculations associated with this type of target function is:

$$K*[N-m(K+1)/2]$$

iterations. When N is relatively large, it is much smaller than C_N^2 . Of course, this type of method is still not the most economical method. However, with regard to when N≤50, the method is realistically feasible. Moreover, it is possible to guarantee obtaining quite a large number of sets of accurate data samples. Thus, it is possible to select from among them relatively good estimations associated with ao and bo.

In the ordinary course of events, ao and bo which are obtained in this way are also capable of being further refined using M estimations. However, due to the fact that our purpose is not to find the most precise ao and bo but is to delete outliers, as a result, we did no further refining. Practical realization demonstrated that ao and bo which are solved for in this way are already quite effective with respect to deleting outliers.

III. DELETION OF OUTLIERS

{ΔTi}, {Δθi} opt for the use of linear models which are shown in equation (7). Using the methods described in sections above, we are then able to obtain robust estimates for ao, a1, bo, b1 /116 Because of this, methods associated with the deletion of outliers can be summarized as:

1. Random Outlier Deletion

Calculation residual errors:

$$\left.\begin{array}{l} r_{Ti} = \Delta T_i - [a_0 + a_1(t_i - t_0)], \\ r\theta_i = \Delta\theta_i - [b_0 + b_1(t_i - t_0)], \quad i = 1,2,\cdots\cdots N, \end{array}\right\} \tag{11}$$

delete

$$|r_{Ti}| > 0.5^s,$$
$$|r\theta_i| > 0.03^\circ$$

outlier values.

2. Deletion of System Outlier Values

System errors given rise to by timing or instrument reading drift or jumps as well as errors given rise to by observations of erroneous targets will in all cases be reflected in ao, a1, bo, b1.

- ao reflects time system differences as well as errors along tracks given rise to because of initial orbital inaccuracies;
- a1 reflects satellite speed errors, including errors given rise to by inaccuracies in satellite periods, eccentricities e, and perigee angles ω;
- bo reflects errors associated with orbital ascending node longitude Ω as well as errors given rise to because of surface height imprecision (corresponding to satellite geocentric range r);
- b1 reflects angle of inclination errors as well as errors given rise to by r (corresponding to a, e, and ω) inaccuracies.

Due to initial orbital errors, influences on ao are very large. We cannot use ao obtained by single station ring data in order to precisely identify time system errors. As a result, we did not use ao in order to delete system outliers.

Due to the fact that our initial orbits had a certain precision, as a result, we are able to use a1, bo, and b1 in order to delete system outliers. The method is as follows.

Delete
$$|b_0| > 0.3^\circ$$
$$|b_1| > 0.1^\circ/\text{minute}$$
$$|a_1| > 2\text{s/minute}$$

station rings.

These criteria are given on the basis of initial orbit

precisions which we are capable of obtaining as well as through experimental test calculations of orbits. Below, we will discuss these further.

IV. CALCULATION EXAMPLES

The radicals for a certain satellite on 27 September 1990 $0^{h}0$ are:

$a = 1.09776422$ (RE) $\quad e = 0.004334$
$i = 82.°584$ $\quad \xi = 103.°507$
$\Omega = 280.722$ $\quad \lambda = 68.°672$
$P = -0.0004$ minutes/celestial ring

As far as a certain survey station on 26 and 27 September is concerned, a ring of various data obtained is as follows. /117

26 September

	T			A	h
	h	m	s		
1	14	9	45.68	277°0'	17°29'
2	14	9	56.26	280°10'	17°46'
3	14	10	25.48	288°46'	18°09'
4	14	10	30.12	290°09'	18°10'
5	14	10	34.94	291°36'	18°09'
6	14	10	39.01	292°46'	18°08'
7	14	10	44.27	294°22'	18°05'
8	14	10	49.84	296°00'	18°01'
9	14	11	40.51	310°18'	16°30'
10	14	11	44.93	311°27'	16°18'

27 September

	T			A	h
	h	m	s		
1	22	57	46.12	35°21'	17°59'
2	22	57	53.10	37°07'	18°36'
3	22	58	28.12	46°52'	21°19'
4	22	58	35.85	49°19'	21°50'
5	22	58	45.36	52°24'	22°26'
6	22	59	03.10	58°28'	23°20'
7	22	59	19.21	64°16'	23°56'
8	22	59	38.59	71°24'	24°14'
9	22	59	53.23	76°52'	24°09'
10	23	00	10.95	83°22'	23°44'
11	23	00	20.51	86°47'	23°23'
12	23	00	37.19	92°32'	22°32'
13	23	00	58.96	99°25'	21°06'
14	23	01	21.18	105°46'	19°24'
15	23	01	46.56	112°05'	17°18'
16	23	02	11.23	117°22'	15°12'

During test calculations, ΔT and Δθ sequences obtained from calculations associated with 26 September data points 1,2,3,4, time additions of 0.7s and 27 September data points 1,2,5,7,15 A plus 20' and points 3,9 A minus 20'are as follows:

26 September

	ΔT(min)		Δθ(deg.)
1	-0.0009	*	0.0122
2	-0.0026	*	0.0113
3	-0.0058	*	0.0015
4	-0.0061	*	-0.0043
5	0.0057		-0.0002
6	0.0028		-0.0032
7	0.0045		-0.0019
8	0.0031		-0.0036
9	-0.0014		-0.0086
10	-0.0023		-0.0067

/118

27 September

	ΔT(min)		Δθ(deg.)	
1	0.0083	*	0.0500	*
2	0.0114	*	0.0430	*
3	-0.0189	*	-0.0312	
4	-0.0028		0.0026	
5	0.0127	*	0.0160	
6	-0.0017		-0.0031	
7	0.0140	*	-0.0036	
8	-0.0026		-0.0085	
9	-0.0169	*	-0.0016	
10	-0.0017		-0.0011	
11	-0.0017		-0.0055	
12	0.0003		-0.0041	
13	-0.0002		-0.0007	
14	0.0029		0.0052	
15	0.0044		0.0069	
16	0.0214	*	-0.0437	*

Explanation: ΔT and Δθ sequences which have * after them were deleted during calculations.

In the calculation examples above, error in data observations is $\sigma\sim$ 2'. Moreover, artificially added errors are approximately 20' ($\sim 10\sigma$). Calculation results clearly show that the methods in question are all capable of correctly deleting them. As a result, methods are successful. Moreover, we also have no difficulty in seeing that the breakdown points associated with the methods in question are already approaching 50% (First ring error data accounted for 40%. Second ring error data accounted for 43.75%.).

V. DISCUSSION

As far as outlier deletion in artificial satellite observation angles is concerned, the problems which it involves are very numerous--for example, the selection of robust estimation methods, how much initial information is required by methods, the level of breakdown points, and so on. However, the most basic problems are:

1. requirements with regard to initial orbit precision;
2. rational selection of deletion criteria.

These two problems are closely related. Large amounts of test calculations clearly show that:

1. the smaller given criteria are--that is, our requiring the deletion of relatively small outlier values--the more accurate are the initial orbital precisions required by methods;
2. when initial orbital precisions are relatively bad, it is easy to give rise to phenomena associated with the irrational deletion of entire rings, that is, deletion of a1, bo, and b1 is not due to observation system error, but is given rise to because of intial orbit errors.

The appearance of these situations can be understood. We are only able to give deletion criteria on the basis of the actual status of initial orbital precision. In situations associated with transit theodolite observations, errors in observations are approximately 2'-3'. Due to the fact that data used because of satellite intermitence periods as well as orbital measurements is also very limited, e and Δ determination errors associated with satellites are relatively large--in particular, when periods of visibility have just begun. As a result, when we select criteria, it is believed that e has $\Delta e \sim 0.002$ and ω has errors of 30° or more.

Test calculations clearly show that, for the criteria given in this article, it is possible to permit initial orbits to have errors this large. Moreover, when there are only ascending phase (or descending phase) observations, initial orbit errors can also be somewhat larger--for example, ω can have errors of around 90° /119 . When initial orbit accuracies are inadequate--for instance, when targets have just been acquired--obviously, it is only possible to initially not carry out outlier deletion.

For the sake of appropriately expanding the range of applications of methods--during the selection of criteria--we intentionally make suitable relaxations of deletion criteria. Besides this, when observations show the appearance of plate jump (for example, 10 individual pieces of data have 6 pieces of data that show the appearance of system errors and the number of unusual values exceeds 50% breakdown points) the occurence of erroneous deletions will also happen. The appearance of these situations makes it so that there are still erroneous data (errors relatively small) which have not been deleted during orbital improvements after preprocessing. Among data which has been deleted out, there will also be data which was in fact correct. However, in the final

analysis, the proportion of unusual values is relatively small. As far as their processing methods are concerned, we will study them in detail in another article.

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